Divergence → Vector lines spreading out

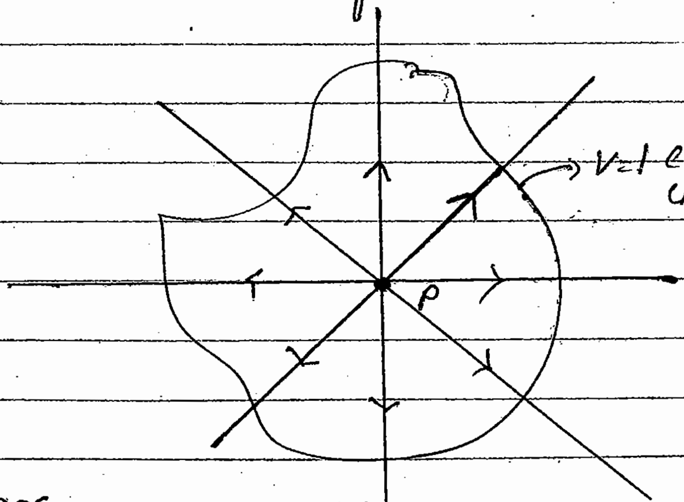
* Divergence of a vector field :-

$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \text{scalar}$$

Divergence of a vector field $\vec{V}$ at any point of $P(x,y,z)$ is defined as the outwards flux of the vector field per unit volume inclose by an infinitisimly close volume inclose by an infinitisimly close surface sorrounding point of P.

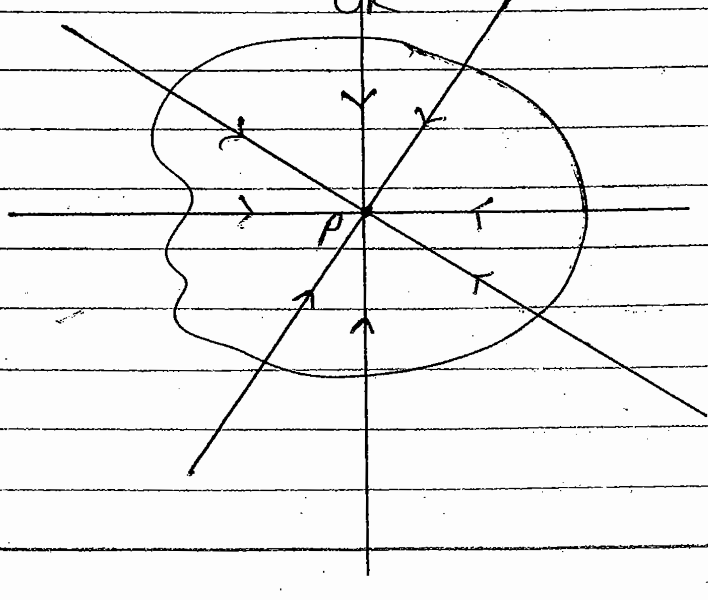
Outward (+ve) ϕ



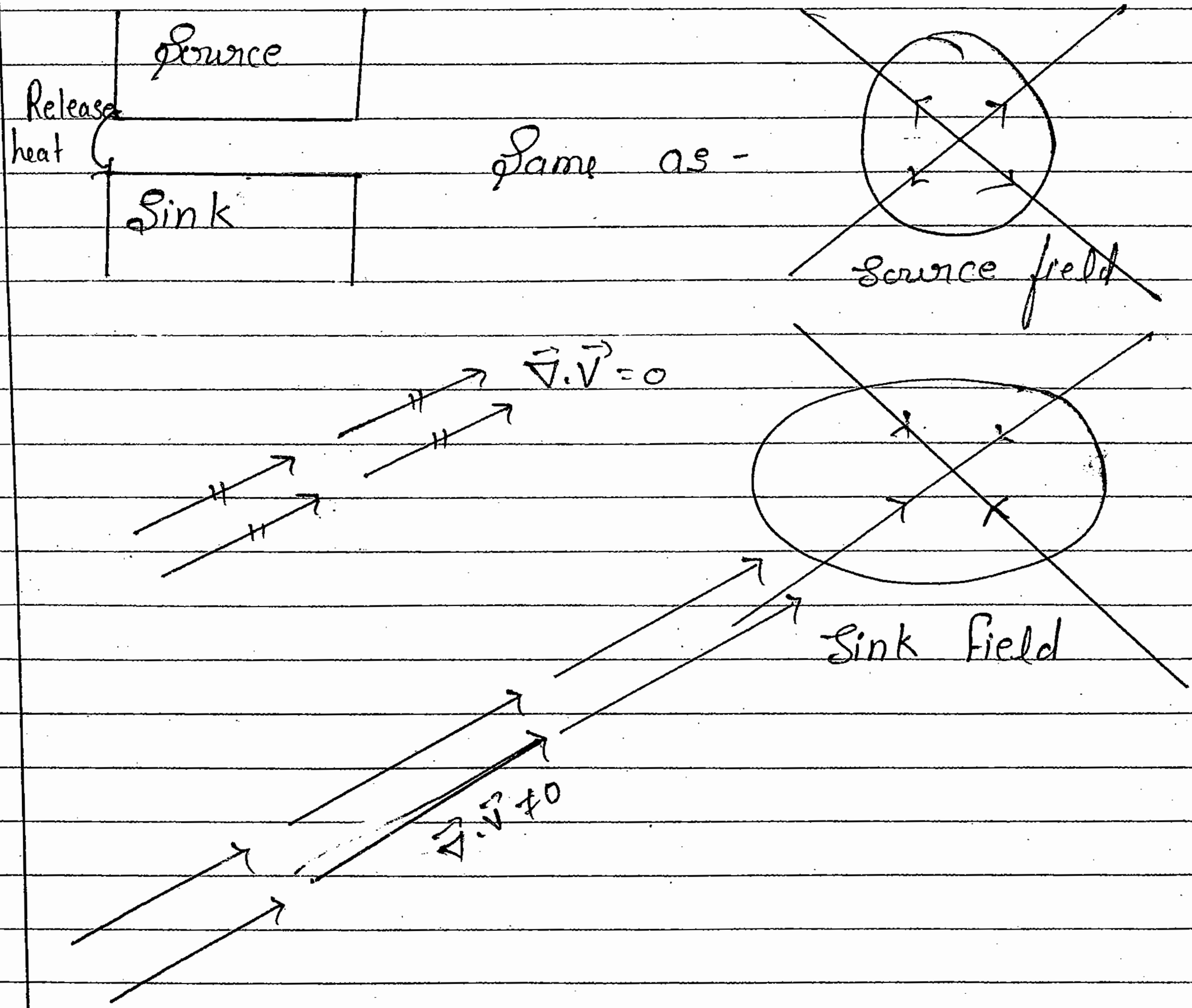
Outward flux per unit volume.

OR

inward (−ve) ϕ.



* $\vec{\nabla}.\vec{V} = +ve \Rightarrow$ Outward flux {Vector lines are going outwards}
* $\vec{\nabla}.\vec{V} = -ve \Rightarrow$ Inward flux [Vector lines coming in or sink].
* $\vec{\nabla}.\vec{V} = 0 \Rightarrow$ Either no flux or net flux is zero. or Solenoidal field.



* If the length or magnitude of the vector lines is changing along a direction along which the vector field is directed out then the vector field has a non zero divergence.

$$\vec{V} = \underline{V_1 \hat{i}} + \underline{V_2 \hat{j}} + \underline{V_3 \hat{k}} \Rightarrow \vec{\nabla} \cdot \vec{V} = 0$$

Constant vector field

Independent of x, y, z

$$\Rightarrow |\vec{V}| = \sqrt{V_1^2 + V_2^2 + V_3^2}$$

Second Condition :-

$$\vec{V} = xy^2 \hat{i}$$

$$|\vec{V}| = \sqrt{}$$

$$\vec{\nabla} \cdot \vec{V} \neq 0$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{U} + \vec{V}) = \vec{\nabla} \vec{U} + \vec{\nabla} \cdot \vec{V}$$

$$\Rightarrow \vec{\nabla} \cdot (u\vec{V}) = (\vec{\nabla} u)\vec{V} + u(\vec{\nabla} \cdot \vec{V})$$

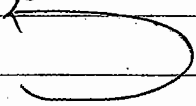
* Curl of a Vector field :-

$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

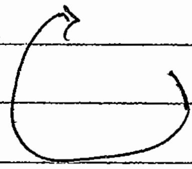
$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

⇒ The magnitude of curl of vector field is the measurement of rotation of the vector field. (Means rotation of vector lines corresponding to vector field)

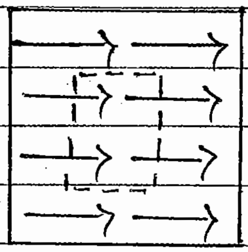
⇒ The direction of curl of a vector field is along the axis of rotation



Direcⁿ of curl = outward $\perp^{or}$ to the paper

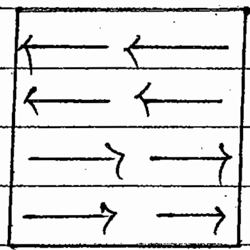


Inward $\perp^{or}$ to the paper

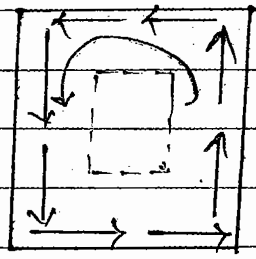


$\vec{\nabla} \times \vec{V} = 0$

(becoz no rotation)



$\vec{\nabla} \times \vec{V} \neq 0$



$\vec{\nabla} \times \vec{V} \neq 0$

⇒ $\vec{\nabla} \times (\vec{U} \times \vec{V}) = \vec{\nabla} \times \vec{U} + \vec{\nabla} \times \vec{V}$

⇒ $\vec{\nabla} \times (u\vec{V}) = (\vec{\nabla} u) \times \vec{V} + u(\vec{\nabla} \times \vec{V})$

⇒ $\vec{\nabla} \times \vec{V} = 0$ ⇒ Irrotational field or Conservative field.

⇒ $\boxed{\vec{V} = \vec{\nabla} \phi} \rightarrow$ Calculate

$\vec{V}$ given, gradiant of scalar

$\vec{\nabla} \times \vec{E} = 0$ ($\vec{E}$ given)

$\vec{E} = -\vec{\nabla} V$ (V Calculate)